*Int. J. of Applied Mechanics and Engineering, 2022, vol.27, No.4, pp.40-51*
DOI: 10.2478/ijame-2022-0049

# INFLUENCE OF FORM DEFECT ON THE MECHANICAL BEHAVIOR AND STRESS INTENSITY FACTOR OF SHRINK-FITTED THICK-WALLED CYLINDERS

Hamid BOUTOUTAOU*
Laboratory of Energetics, Mechanics and Engineering, Faculty of Technology, University M'hamed Bougara, 35000 Boumerdes, ALGERIA
E-mail: h.boutoutaou@univ-boumerdes.dz

Brahim CHEBBAB
Motor Dynamics and Vibroacoustics Laboratory, Faculty of Technology, University M'hamed Bougara, 35000 Boumerdes, ALGERIA

Abdelmounaime BELMESSOUS
Laboratory of Energetics, Mechanics and Engineering, Faculty of Technology, University M'hamed Bougara, 35000 Boumerdes, ALGERIA

In this research work, the finite element software, ABAQUS is used to study by simulations the influence of form defect on mechanical behavior of a shrink-fitted assembly presenting internal radial cracks. Under the action of contact pressure induced by the tightening between two cylinders, these cracks resulting from incorrect assembly operations or materials elaboration defect, can be harmful to the assembly. Various simulations were carried out in two modeling cases, taking into account the geometric parameters of defect (amplitude Df), of cylinders (thickness t) and of cracks (length a, ratio a/t). Another important parameter such as the tightening was also considered in the modeling. The first modeling relates to the case with defect, external cylinder presents an oval (elliptical) form defect and internal radial cracks. The other concerns the perfect equivalent case (without form defect). The comparison of results obtained by two models shows that form defect modifies the uniformity of equivalent stresses distribution in cylinders and increases the value of stress intensity factor (SIF) KI in cracks. Defect amplitude and tightening significantly influence the value of equivalent stress and that of stress intensity factor (SIF) $K_I$ .

**Key words:** shrink-fit, thick-walled cylinder, finite elements, stress intensity factor, crack cylinder.

## 1. Introduction

Shrink-fit is an assembly technique by tightening two parts of generally cylindrical form. The tightening caused by a difference in diameters induces a pressure at interface contact, which keeps the parts together thanks to friction. Under the action of contact pressure, the presence of any micro-cracks resulting from defects in materials elaboration or mishandling during the assembly operation, can damage the parts in contact and thus affect safety and reliability of assembly. This assembly process is increasingly used in several industrial fields, such as mechanical industry, aeronautics, oil drilling, automotive. This is the reason why taking into account the form defect has been integrated in various modeling and simulations in order to analyze its influence on the mechanical behavior of shrink-fitted assemblies presenting cracks in the internal surface. Calculation methods for shrink-fitted assemblies are still traditional and have not changed much for several years. They are based on the thick-walled cylinder theory with internal pressures developed by Timoshenko [1]. This model is limited to elastic behavior (weak tightening) and to simple cylindrical parts whose contact surfaces are assumed to be perfectly smooth [2]. Most of the work carried out

* To whom correspondence should be addressed

in the field of shrinking does not take into account the form defect of shrink-fitted parts. No model considering this defect is currently available. Giiven [3] has developed an analytical solution for a shrink-fitted assembly with an elasto-plastic hub presenting a variable thickness. This analysis is based on Tresca's condition of elasticity, its associated flow rule and its linear work hardening. Özel *et al.* [4] studied the distribution of stresses and strains in shrink-fitted assemblies by varying the joint geometry. It was concluded that the hub geometry has a significant effect on the stresses state and in particular at the level of the contact surface edges. Fontaine and Siala [5] studied in the elastic domain the influence of form defect on local stress state at interface contact. This study showed that the form defect has an influence on mechanical characteristics and strength of assembly. Laghzale *et al.* [6] were interested in the analytical analysis of interference effect on the residual stresses distribution in a shrink-fitted assembly. The results of this analysis showed a significant effect of tightening value on residual stresses state and a good agreement with the results obtained by FEM. An analytical formulation to evaluate the evolution of contact pressure in shrink-fitted joints subjected to a high temperature regime has been proposed by Esposito *et al.* [7]. The formulation gives a clear correlation between contact pressure degradation and creep properties of shrink-fitted materials. The method is relatively simple but reliable for the evaluation of transmission loads in shrink-fitted assemblies subjected to a creep regime.

Regarding the calculation of stress intensity factors (SIFs) in thick-walled cylinders, researchers who have studied different types of cracks and the influence of geometric parameters have carried out many studies. Kirkhope *et al.* [8] used the finite element method for the calculation of mode $I$ stress intensity factors $K_I$, in the case of cylinders under internal pressure containing single and multiple uniform semi-elliptical axial cracks. The effect of cylinder diameter ratio and crack depth on the stress intensity factors was taken into account. Fatigue life prediction taking into account the effect of residual stress distribution by SIFs, has been proposed by Bahloul *et al.* [9] who developed an improved calculation model. The finite element model for the calculation of elastic stress intensity factors and crack opening displacements in a cracked cylinder through the inclined axial wall subjected to internal pressure has been used by Huh *et al.* [10]. This model and the analysis procedure have been validated against existing solutions. Al-Moayed *et al.* [11] calculated the stress intensity factors $K_I$ using the ANSYS Finite Element Software for semi-elliptical circumferential cracks on a thick cylinder. The cracks examined were located either on the outer surface or on internal cylinder and subjected to two different types of loads, traction and internal pressure, applied separately. To present the results in a more complete form, a dimensionless analysis is used and a broad limit of variation of the parameters that define geometry of the crack is considered. They found that the SIF distributions are symmetrical along the crack forehead and a significant effect of crack depth on the value of stress intensity factors (SIF). Predan *et al.* [12] applied the finite element method to solve the problem of a hollow cylinder exhibiting a semi-elliptical crack under pure torsional loading. A general weight function to evaluate the thermal stress intensity factors of a circumferential crack in cylinders is used by Nabavi and Ghajar [13]. This weight function is valid for a wide range of thin and thick-walled cylinders. The results obtained using the finite element method have been compared to existing solutions and indicate excellent agreement. Jun Ying *et al.* [14] used the ABAQUS finite element software and analytical equation taking into account the ratio $\alpha / t$ of crack length on wall thickness, to evaluate the stress intensity factors. The calculation using the finite element software ANSYS of the three modes $K_I, K_{II}$ and $K_{III}$ of internal and external cracks of semi-elliptical shape, located on a thick cylinder subjected separately to bending and torsion, was carried out by Al-Moayed *et al.* [15]. Several geometrical parameters such as the ratio of crack depth to the crack length, the ratio of crack depth to the cylinder thickness $t$ and the ratio of internal radius to cylinder thickness, were taken into account. The analysis and comparison of results obtained from three modes under the two loadings were made.

Currently, most of research work carried out on the study of cracks in thick-walled cylinders focuses on constant pressure loads on the non-contact inner surface. However, in shrink-fitted thick-walled cylinders the cracks appear on the inner contact surface of the outer cylinder. In addition, these cylinders may have form defects resulting from their manufacture, their handling and transport or from their assembly manipulation. In this work, we used the finite element software ABAQUS to study by simulations the influence of oval (elliptical) form defect on the mechanical behavior and stress intensity factor (SIF) $K_I$ of shrink-fitted thick-walled cylinders. Several geometric parameters of form defect and cracks were taken into account. The results obtained in the case with defect are compared with those of an equivalent case (without defect).

## 2. Theoretical analysis

### 2.1. Stresses and pressure in shrink-fitted thick-walled cylinders

As shown by Eqs. (2.1)-(2.2) and (2.5)-(2.6) of thick-walled cylinders theory and Lame's equations [16], when the parts are perfectly cylindrical, the principal stresses at the cylinders interface are composed of radial compression $\sigma_r$ and circumferential $\sigma_\theta$ in the inner cylinder and radial compression and circumferential tension in the outer cylinder.
For the inner cylinder we have

$$\sigma_r = -p_f \frac{r_1^2}{r_1^2 - r_0^2}\left(1 - \frac{r_0^2}{r^2}\right), \tag{2.1}$$

$$\sigma_\theta = -p_f \frac{r_1^2}{r_1^2 - r_0^2}\left(1 + \frac{r_0^2}{r^2}\right), \tag{2.2}$$

and a Von Mises equivalent stress

$$\sigma_{VM} = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_r - \sigma_\theta\right)^2 + \left(\sigma_r\right)^2 + \left(\sigma_\theta\right)^2}\ , \tag{2.3}$$

which give

$$\sigma_{VM} = p_f \frac{r_1^2}{r_1^2 - r_0^2}\sqrt{3 + \left(\frac{r_0}{r}\right)^4}\ . \tag{2.4}$$

For the outer cylinder we have

$$\sigma_r = p_f \frac{r_1^2}{r_2^2 - r_1^2}\left(1 - \frac{r_2^2}{r^2}\right), \tag{2.5}$$

$$\sigma_\theta = p_f \frac{r_1^2}{r_2^2 - r_1^2}\left(1 + \frac{r_2^2}{r^2}\right), \tag{2.6}$$

and a Von Mises equivalent stress is

$$\sigma_{VM} = p_f \frac{r_1^2}{r_2^2 - r_1^2}\sqrt{1 + 3\left(\frac{r_2}{r}\right)^4}\ , \tag{2.7}$$

with $p_f$ the contact pressure. For two cylinders of the same materials we have:

$$p_f = \frac{\delta}{r_1\left[\frac{1}{E}\left(\frac{r_1^2 + r_0^2}{r_1^2 - r_0^2} + \frac{r_2^2 + r_1^2}{r_2^2 - r_1^2} - 2\nu\right)\right]} \tag{2.8}$$

where, $r_0, r_1$ and $r_2$, the cylinders inner and outer radii, $\delta$ the tightening between cylinders and $E, \nu$ mechanical characteristics of the two cylinders materials. This stress state gives an equivalent Von Mises stress greater than four times the interface pressure (Eqs (2.4) and (2.7)). The introduction of the form defect changes the classic notion of tightening because the minor axis *bi* of the outer cylinder then varies at each point, Fig.1. Two definitions of it can then be introduced. The maximum tightening $\Delta_M$ and the mean tightening

$$\begin{aligned} &\Delta_M = r_1 - b_i \ , \\ &\Delta_m = r_1 - r_{eq} \ , \end{aligned} \tag{2.9}$$

with $r_1$ outer radius of the inner cylinder, *bi* minor axis of the oval outer cylinder and $r_{eq}$ the inner radius of the equivalent outer cylinder. By equalizing the interior surfaces of the two oval and equivalent cylinders, Eq.(2.10), we obtain:

$$\begin{aligned} &\pi r_{eq}^2 = \pi a_i b_i \ , \\ &r_{eq} = \sqrt{a_i b_i} \ . \end{aligned} \tag{2.10}$$

(a) (b)

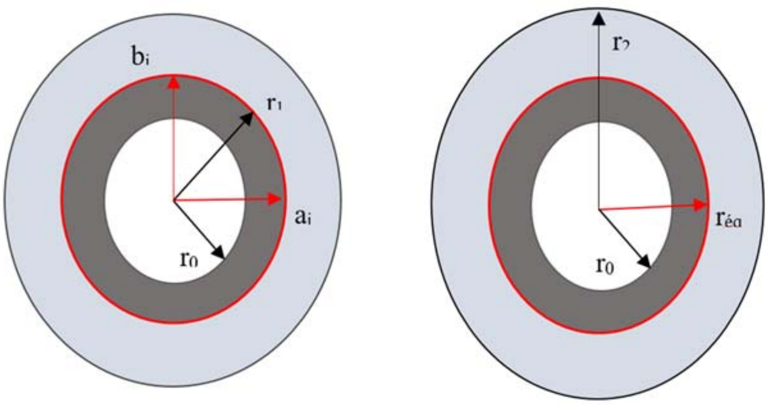



Fig.1. The assemblies studied: defect case (a), equivalent perfect case (b).

**2.2. Stress intensity factor for thick-walled cylinders**

As we have shown previously, in the outer cylinder the contact pressure causes, a radial compression $\sigma_r$ and a circumferential tension $\sigma_\theta$. In the case of the presence of a radial crack, this tension tends to open and propagate it, which corresponds to mode $I$, of crack opening. Calculation of the mode $I$ stress intensity factor $K_I$ in cylinders with thick walls is generally calculated [17] by Eq.(2.11).

$$K_I = p\sqrt{\pi a}\, f\left(a/_t\right) \tag{2.11}$$

where $p$ is the contact pressure without crack, $f\left(a/t\right)$ is a correction factor which depends on the ratio of the length (depth) of crack *a* to the thickness *t* of the wall. In the case of an oval or elliptical form cylinder, this relation is no longer valid. The only way to calculate $K_I$ is numerically, using the finite elements method. The stress intensity factor $K_I$ does not only depend on $a/t$, but also on the cylinder form defect $D_f$.

## 3. Modeling and simulation by finite elements

### 3.1. Simulation of shrink-fitted cylinders with consideration of the defect

The shrink-fit assemblies studied are made up of two cylinders of the same materials (steel) and thickness $t = 20\,mm$. The perfect inner cylinder with respective radii $r_0 = 60\,mm$, $r_1 = 80\,mm$ and the outer cylinder of elliptical (oval) form with an amplitude $D_f = 0.15\,mm$, interior main axes $a_i = 80\,mm$ and $b_i = a_i - D_f = 79.85\,mm$.

#### 3.1.1. Meshes and boundary conditions used

In various modeling and simulations carried out, finite elements CPS8R type were used. The element size is $(0.5\,x\,0.5)\,mm$ for both cylinders, Fig.2a. The boundary conditions applied correspond to blockages in rotation and in translation along *x* of cylinders vertical part (Fig.2b) and along *y* of horizontal part (Fig.2c).

(a) (b) (c)

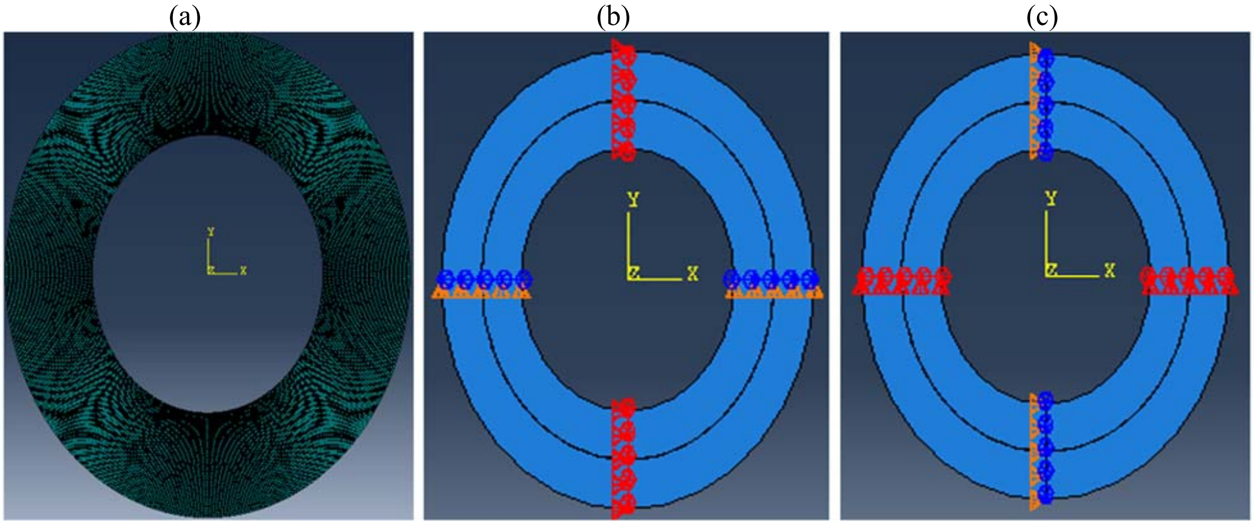



Fig.2. Cylinder mesh (a) and boundary conditions applied (b), (c).

#### 3.1.2. Results and discussions

Figure 3 shows the Von Mises stress distribution nephogram in two simulated cases, equivalent perfect case (a) and case with form defect $D_f = 0.15mm$ (b). One easily notes the effect of defect on distribution and value of Von Mises equivalent stress in the two cylinders. The comparison of Von Mises stress, principal stresses and contact pressure variation at cylinders contact interface for two modeling cases is given in Fig. 4. The stresses are constant along the interface in the case of perfect cylinders (a). On the other hand, they pass through a minimum and a maximum stress in the case with defect. We also notice that tension stress in the case with defect is significant (b). These results clearly show that the presence of form defect changes the uniformity of stress distribution and can affect the mechanical behavior of assembly.

| (a) | (b) |
|---|---|

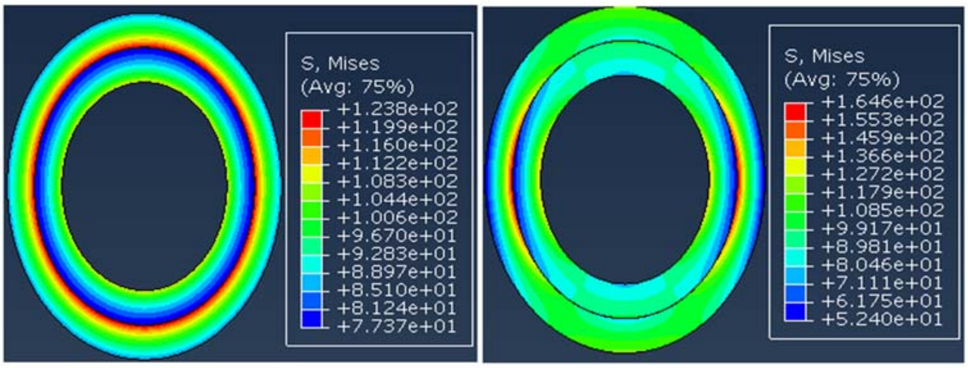
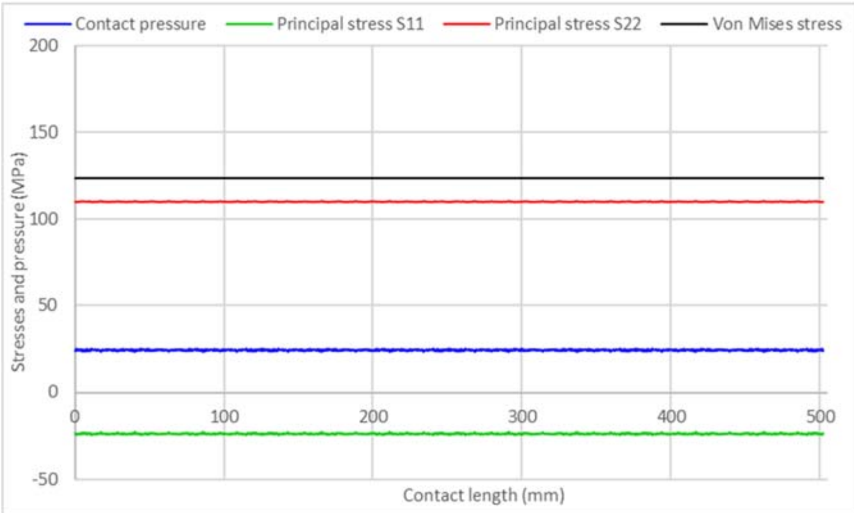



Fig.3. Von Mises stress distribution nephogram: equivalent case (a), form defect case (b).

(a)




(b)

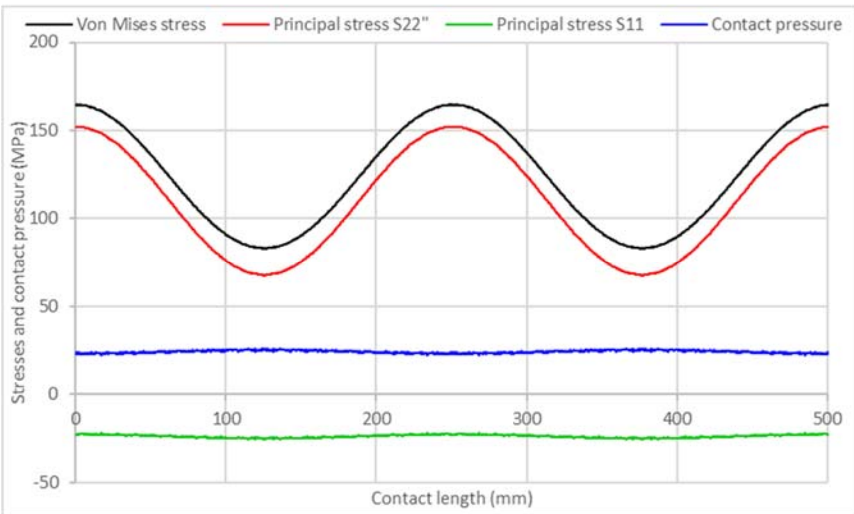



Fig.4. Von Mises stress, principal stresses and contact pressure: form defect case (a) and equivalent case (b).

### 3.2. Simulation of fretted cylinders with cracks taken into account

In next simulations, we used same cylinders characteristics. To study better the form defect effect on the behavior of cracks, we took into account the following parameters: defect amplitude $Df:50-250\,\mu m$, ratio $a/t:0.125-0.25$, thickness $t:10-20\,mm$ and mean tightening : $25-100\,\mu m$. Various radial cracks are located on the internal contact surface of the outer cylinder where the stress is maximum.

#### 3.2.1. Finite elements model and mesh

Figure 5 (a) and (b), shows mesh and $2D$ finite elements model used. Finite elements CPS8R and CPS6 types are used respectively for cylinders mesh and crack tip. The method calculating stress intensity factor (SIF) $K_I$ is the contour integral method, the midside node parameter equal to $0.25$, the number of contours equal to $5$, and the crack initiation criterion is the maximum tangential stress.

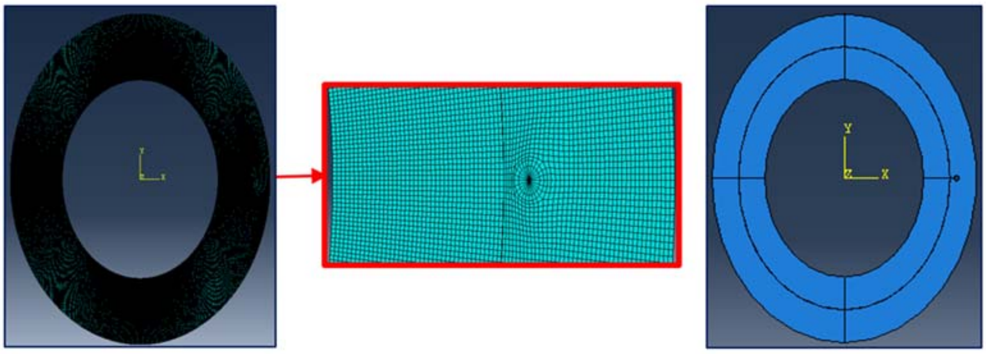



Fig.5. Mesh and $2D$ finite elements model.

#### 3.2.2. Results and discussions

Figure 6 shows the Von Mises stress distribution nephogram for a crack of length $a=3\,mm$, *defect* $D_f=0.15\,mm$ and mean tightening $\Delta_m=0.075\,mm$, in both cases of simulation. The presence of a crack changes the mechanical behavior of assembly, especially in the presence of form defect. The maximum value of the Von Mises stress is located at the tip crack, it is $301.3\,MPa$ for the case with defect and $233.1\,MPa$ for the equivalent case.

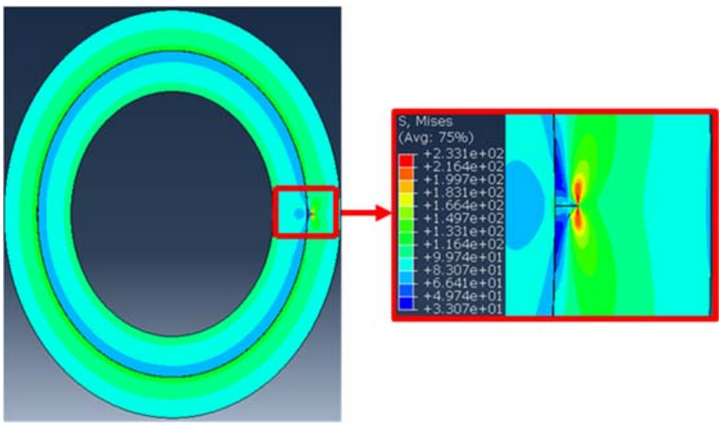



Fig.6a. Von Mises stress distribution nephogram: equivalent case.

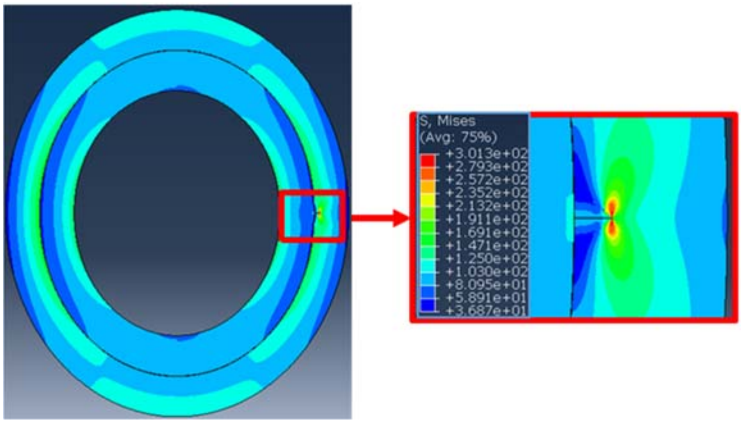



Fig.6b. Von Mises stress distribution nephogram: form defect case.

The distribution of principal Von Mises stresses and contact pressure is shown in Fig. 7. Compared to the case with defect and without crack, one notes at the beginning of crack a significant decrease in the Von Mises stresses, principal $S_{22}$ and a remarkable increase in principal stress $S_{11}$ and contact pressure.

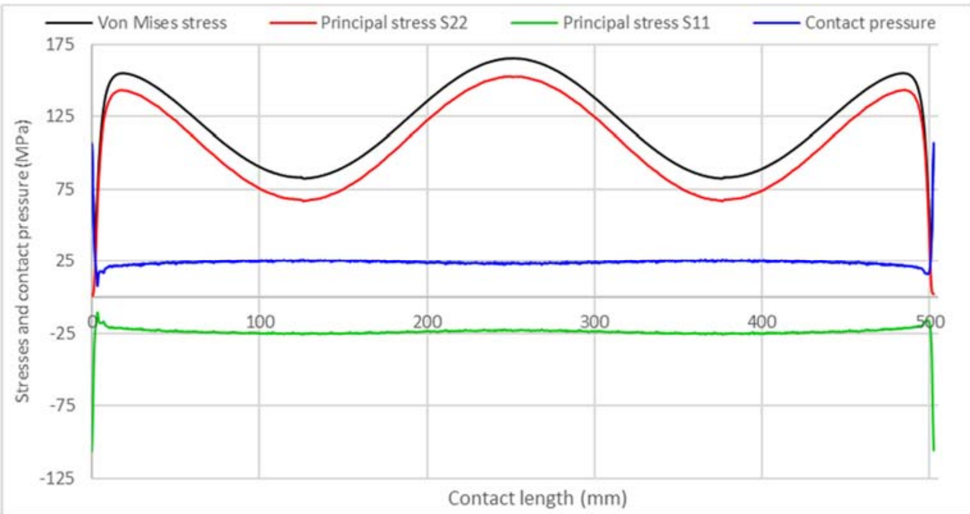



Fig.7. Von Mises stress, principal stresses and contact pressure for the case with form defect and a crack.

In the analysis of results that follows, we compared two simulation cases. Figure 8 shows the influence of form defect amplitude on mechanical behavior and the value of SIF $K_I$. The case with defect is the most unfavorable, when defect amplitude increases, the difference in values between two cases increases considerably. For a defect of $0.25\,mm$, the outer cylinder reaches plasticity.

The effect of crack length is shown in Fig.9. Crack length $a$ varies from $2.5-5\,mm$ and $a/t$ from $0.125-0.25$. When the crack is deep, stress intensity factor values and other quantities increase with a small difference between the different values.

In Fig. 10, the influence of cylinders thickness $t\left(t=10-20\,mm\right)$ on different quantities is weak for the form defect case and almost zero in the equivalent case.

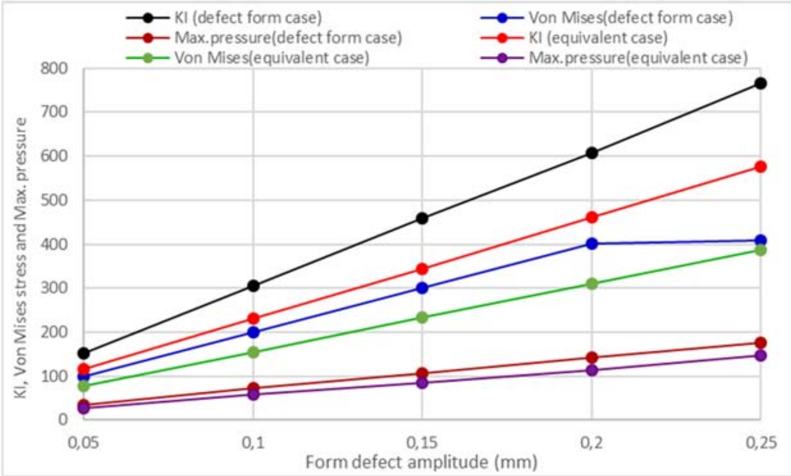



Fig.8. Effect of form defect amplitude on Von Mises stress, max. pressure and SIF $K_I$ in both cases.

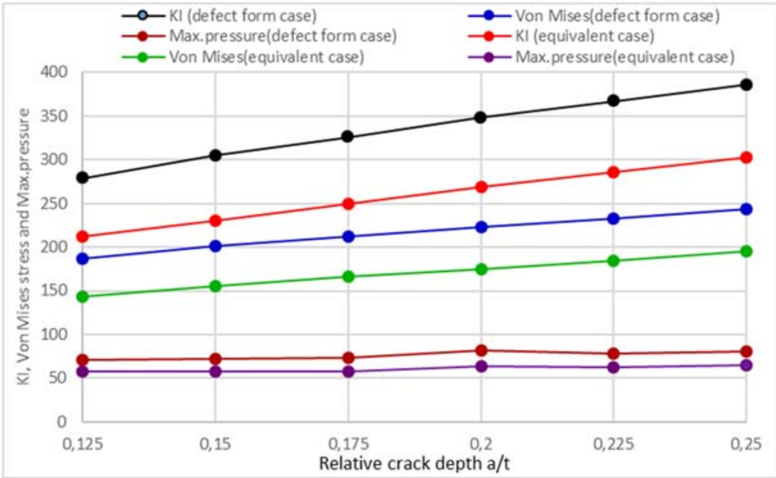



Fig.9. Variation of Von Mises stress, max. pressure and the SIF $K_I$ as a function of $a/t$ ratio in both cases.

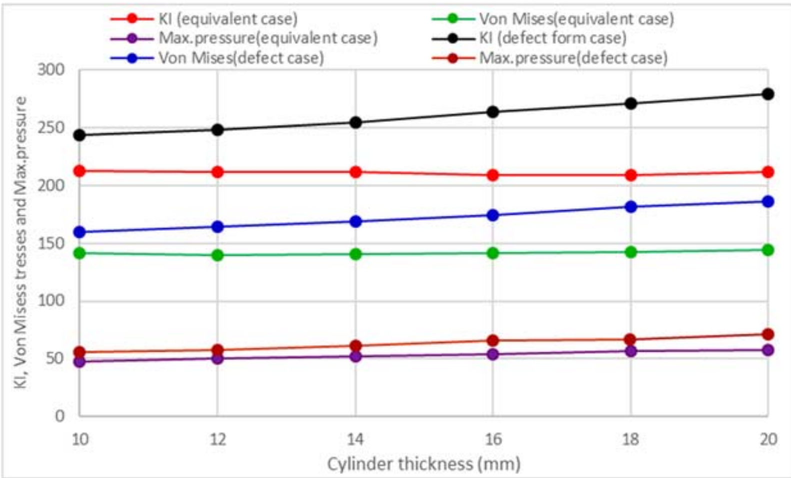
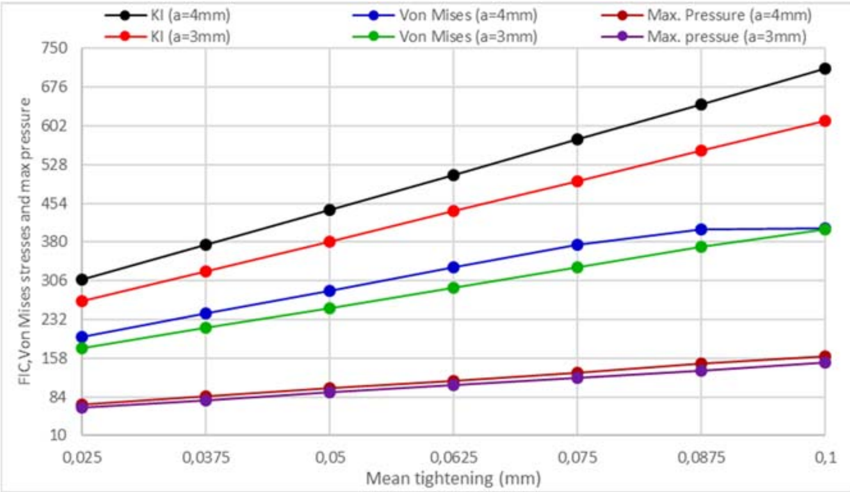
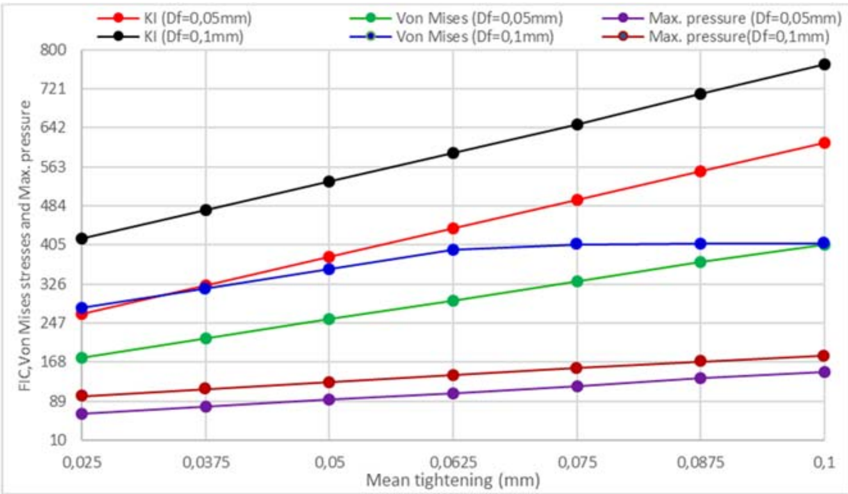



Fig.10. Variation of Von Mises stress, max. pressure and SIF $K_I$ as a function of cylinders thickness $t$ in both cases.

To study better the effect of the tightening parameter on assembly behavior, we considered two different simulations. In the first, $a = 3mm, a = 4mm$ and $D_f = 0.05mm$, fig. 11(a). In the other, $a = 3mm$ and $D_f = 0.05mm, D_f = 0.1mm$, Fig. 11(b). Equivalent cases were not simulated. The results show that the tightening has a significant effect on different quantities values, but the most noticeable is that the variation of these quantities in two simulations is clearly different. The form defect effect is most dominant compared to that of crack length. It is also observed that the outer cylinder reaches plasticity. It is faster for $D_f = 0.1mm$ than for $a = 4mm$.




(a)




(b)

Fig.11. Influence of tightening average on Von Mises stress, max. pressure and SIF $K_I$: $D_f = 0.05mm$ and $a = 3mm, a = 4mm$ (a); $a = 3mm$ and $D_f = 0.05mm, D_f = 0.1mm$ (b).

## 4. Conclusion

We have shown in this study that it is not only possible but essential to integrate the form defect in the modeling of shrink-fitted assemblies. If the classic calculation models do not allow it because of their simplifying assumptions, it is quite possible today thanks to the finite elements method to integrate form defects into a numerical model. Indeed, defects are inherent in the process of elaboration of materials and obtaining mechanical parts. Taking into account the form defect modifies the tension stress and Von Mises stress distribution uniformity at the outer cylinder contact interface. This new state of stress can damage the parts in contact and thus affect the safety and reliability of shrink-fitted assembly, as the results show. The influence of defect amplitude and tightening on values of stresses and stress intensity factor (SIF) is very remarkable compared to that of cracks length (depth). The effect of cylinder thicknesses is small.

**Acknowledgment**

We would like to thank the Director of the Energetics, Mechanics and Engineering Laboratory / Faculty of Technology, University of Boumerdes, Algeria for his help and continuous encouragement to carry out this research work.

## Nomenclature

$\sigma_r$ – radial stress
$\sigma_\theta$ – circumferential stress
$\sigma_{VM}$ – Von Mises stress
$S_{11}$ – radial principal stress
$S_{22}$ – circumferential principal stress
$p_f$ – shrink-fit pressure
$p$ – contact pressure without crack
$r_0$ – inner radii of inner cylinder
$r_1$ – outer radii of inner cylinder
$r_2$ – outer radii of outer cylinder
$r_{eq}$ – equivalent inner radii of outer cylinder
$r$ – variable radii
$a_i$ – major axis of oval outer cylinder
$b_i$ – minor axis of oval outer cylinder
$D_f$ – form defect
$t$ – cylinders thickness
$E$ – Young's longitudinal modulus
$\nu$ – Poisson's ratio
$a$ – crack length
$K_I$ – stress intensity factor
$\delta$ – cylinders tightening
$\Delta_M$ – cylinders maximum tightening
$\Delta_m$ – cylinders mean tightening

## References

[1] Timoshenko SP. (1956): *Strength of Materials. Part 2, Advanced Theory and Problems, Third Ed.*– Van Nostrand, Princeton.

[2] NF E22-620 et E22-622 (1984): *Assemblages frettés sur portée cylindrique*.– AFNOR, Paris la Défense.

[3] Given U. (1991): *The shrink fit with elastic-plastic hub exhibiting variable thickness.*– Acta Mechanica, vol.89, pp.65-72.

[4] Özel A., Temiz S., Aydin M.D. and Şen S. (2005): *Stress analysis of shrink-fitted joints for various fit forms via finite element method.*– Materials and Design, vol.26, pp.281-289.

[5] Fontaine J.F. and Siala I.E. (1998): *Three-dimensional modelling of a shrinkage fit taking into account the Form defects.*– European Journal of Mechanics - A/Solids, Elsevier, vol.17, No.1, pp.107-119.

[6] Laghzale N., Bouzid A. and Elgharad A. (2016): *Analytical modelling of elastic-plastic interference fit joints.*– International Review on Modelling and Simulations (IREMOS), vol.9, No.3, pp.191-199.

[7] Esposito L. and Bruno M. and Bertocco A. (2020): *Analytical formulation of the contact pressure evolution for interference joints under creep regime.*– International Journal of Pressure Vessels and Piping, vol.185, 104126.

[8] Kirkhope K.J., Bell R. and Kirkhope J. (1991): *Stress intensity factors for single and multiple semi-elliptical surface cracks in pressurized thick-walled cylinders.*– International Journal of Pressure Vessels and Piping, vol.47, No.2, pp.247-257.

[9] Bahloul A., Bouraoui C. and Boukharouba T. (2017): *Prediction of fatigue life by crack growth analysis.*– The International Journal of Advanced Manufacturing Technology, vol.91, pp.4009-4017.

[10] Huh N.S., Shim D.J., Choi S. and Park K.-B. (2008): *Stress intensity factors and crack opening displacements for slanted through-wall cracks in pressurized pipes*.– Fatigue & Fracture of Engineering Materials & Structures, vol.31, No.6, pp.428-440.

[11] Al-Moayed O.M. , Kareem A.K. , Ismail A.E. , Jamian S. and Nemah M.N. (2019): *Distribution of mode I stress axial intensity factors for single circumferential semi-elliptical crack in thick cylinder.*– International Journal of Integrated *Engineering.* vol.11 No.7, pp.102-111.

[12] Predan J., Močilnik V. and Gubeljak N. (2013): *Stress intensity factors for circumferential semi-elliptical surface cracks in a hollow cylinder subjected to pure torsion*.– Engineering Fracture Mechanics. vol.105, pp.152-168.

[13] Nabavi S.M. and Ghajar R. (2010): *Analysis of thermal stress intensity factors for cracked cylinders using weight function method.*– International Journal of Engineering Science, vol.48, No.12, pp.1811-1823.

[14] Jun Ying, Zhaojun Yang, Chuanhai Chen, Xiaoxu Li, Hailong Tian (2019): *Stress intensity factors for thick- walled cylinder under interference fit.*– 4th International Conference on System Reliability and Safety *(ICSRS)*, pp.359-363.

[15] Al-Moayed O.M. , Kareem A.K. , Ismail A. E. , Jamian S. , Nemah M. N. (2020): *Influence coefficients for a single superficial cracked thick cylinder under torsion and bending moments.*– International Journal of Integrated Engineering, vol.12, No.4, pp.132-144.

[16] Xu Bingye, Liu Xinsheng. (1995): *Applied Elasticity and Plasticity.*– Tsinghua University Press, pp.236-242.

[17] Kirkhope K. J., Bell R. and Kirkhope J. (1990): *Stress intensity factors equations for single and multiple cracked pressurized thick-walled cylinders.*– International Journal of Pressure Vessels and Piping, vol.41, No.1, pp.103-111.

Received: May 4, 2022
Revised: October 15, 2022